# Harmonic analysis and prediction

A. S. Unnikrishnan

- Tidal variations can be represented as a sum of finite number of harmonics

$H_n \cos(\sigma_n t - g_n)$ $\sigma_n = 360\ \omega_n / 2\ \Pi$

angular speed in degrees per mean solar hour

- The frequencies can be expressed as individual, sum or difference between frequencies.

$\omega_n = i_a\omega_1 + i_b\ \omega_2 + i_c\omega_3 + i_d\omega_4 + i_e\omega_5 + i_f\omega_6$

where ia, ib etc.. are integers of small values (0,1,2 etc…). For example, for M2,

The numbers are 2,0,0,0,0 and 0 (Doodson numbers)

**Table 3.2** Basic astronomical periods and frequencies

| | Period | | Frequency | | Angular speed | |
|---|---|---|---|---|---|---|
| | | | $f$ cycles per mean solar year | $\sigma$ degrees per mean solar hour | Symbol in rate of radians | Rate of change of |
| Mean solar day | 1.00 | mean solar days | 1.00 | 15.0000 | $\omega_0$ | $C_s$ |
| Mean lunar day | 1.0351 | mean solar days | 0.9661369 | 14.4921 | $\omega_1$ | $C_l$ |
| Sidereal month | 27.3217 | mean solar days | 0.0366009 | 0.5490 | $\omega_2$ | $s$ |
| Tropical year | 365.2422 | mean solar days | 0.0027379 | 0.0411 | $\omega_3$ | $h$ |
| Moon's perigee | 8.85 | Julian years | 0.0003093 | 0.0046 | $\omega_4$ | $p$ |
| Regression of Moon's nodes | 18.61 | Julian years | 0.0001471 | 0.0022 | $\omega_5$ | $N$ |
| Perihelion | 20,942 | Julian years | – | | $\omega_6$ | $p'$ |

The frequencies can be expressed as individual, sum or difference between frequencies.

$2\omega_0$ indicates S2 — $\omega_0$ 1 day

$2\omega_1$ indicates $M_2$ — $\omega_1$ 1.03 day

$2\omega_1 - (\omega_2 - \omega_4)$ indicates $N_2$

$2\omega_1 + (\omega_2 - \omega_4)$ indicates L2

(-sign inside the bracket indicates moon's perigee moves backward)

$\omega_1 - \omega_2$ $O_1$ lunar

$\omega_1 + \omega_2$ also $\omega_0 + \omega_3$ $K_1$ is luni solar

# Phase lag (local)

- Local phase (K)  is not much used presently
- K  lag of the phase (epoch) of the particular constituent behind the phase of the corresponding equilibrium constituent at the place

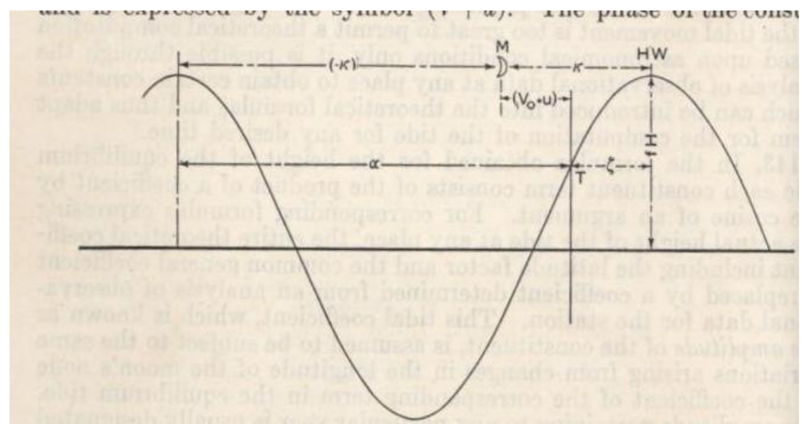



FIGURE 7.

uent itself at this time is reckoned from the preceding high wat and therefore equals $(V+u-\kappa)$.

# Time zone conversion

- $fH \cos(\sigma t - G + V)$
- Where V is called the astronomical argument, G is the phase lag of the constituent with respect to the equilibrium tide on Greenwich longitude.
- gn is the phase lag for time zone
- $G = g + j\sigma$
- Greenwich phase = local time zone phase+ zone shift in hours * constituent speed in degrees per hour
- G is used for co-tidal charts from global tidal models.

# Nodal adjustment

Without nodal adjustment, the amplitude of $M_2$ will increase or decrease by 3.7 percentage.

- $H_n \ f_n \cos[\sigma_n t - g_n + (V_n + u_n)$

  where $V_n$ is the phase angle at time zero

- $f_n$ the nodal amplitude factor
- $u_n$ the nodal angle

  Nodal corrections are done through f and u

They are approximately,

- $f = 1\text{-}0.0373 \cos \omega_5 t$
- And $u = -2.1 \sin \omega_5 t$

- The node factor and nodal angle are 1.0 and 0.0 for solar constituents.

# Analysis of sea level observations

- A time series of sea level observations can be analysed to obtain the amplitudes and phases of major tidal constituents by a least square procedure known as Harmonic analysis.
- This is done by fitting a tidal function with observed $R(t)=X(t)-T(t)-Z_0$ when summed over all the observed values has a minimum value.
- $H_n$, $g_n$ are the unknowns, $f_n$ $u_n$ are the nodal adjustments and $V_n$ is the Equilibrium phase angle or argument (For $M_2$ , it is -2s+2h)

# Calculation of orbital elements

**Table 4.2** The orbital elements used for harmonic expressions of the Equilibrium Tide

| | |
|---|---|
| Mean longitude of the Moon | $s = 218.32 + 481267.88° \times T$ |
| Mean longitude of the Sun | $h = 280.47 + 36000.77° \times T$ |
| Longitude of lunar perigee | $p = 83.35 + 4069.01° \times T$ |
| Longitude of lunar ascending node | $N = 125.04 - 1934.14° \times T$ |
| Longitude of perihelion | $p' = 282.94 + 1.72° \times T$ |

$$T = \frac{(Time - 0.5)}{36525}$$

*T* is in units of a Julian century (36,525 mean solar days).
*Time* is measured in days from 0 hours UT on 0/1 January 2000.
The 0.5 arises because the constants in the five equations refer to midday on 1 January.

# Nodal adjustments

**Table 4.3** Basic nodal modulation terms for the major lunar tidal constituents

| | *f* | *u* |
|---|---|---|
| $M_m$ | $1.000–0.130\cos(N)$ | 0.0° |
| $M_f$ | $1.043–0.414\cos(N)$ | $-23.7°\sin(N)$ |
| $Q_1$, $O_1$ | $1.009–0.187\cos(N)$ | $10.8°\sin(N)$ |
| $K_1$ | $1.006–0.115\cos(N)$ | $-8.9°\sin(N)$ |
| $2N_2$, $\mu_2$, $\nu_2$, $N_2$, $M_2$ | $1.000–0.037\cos(N)$ | $-2.1°\sin(N)$ |
| $K_2$ | $1.024–0.286\cos(N)$ | $-17.7°\sin(N)$ |

$N = 0$ March 1969, November 1987, June 2006, April 2025, September 2043, . . ., at which times the diurnal terms have maximum amplitudes, whereas $M_2$ is a minimum. $M_2$ has maximum Equilibrium amplitudes in July 1978, March 1997, October 2015, March 2034, March 2053, . . .

# Criterion for Separation of tidal constituents

- Raleigh criterion
- gives the minimum number of days of data required to separate any two constituents. It is given by
- 360/ (diff in angular speeds)

For instance, to separate M2 from S2, one requires 360/ (30.0-28.984). The unit is in hours

Ans:

# Inference of constituents

- Many constituents cannot be separated with short duration of data. However, past analyses show that minor constituents are related empirically with major.  For example,  amplitude of K2 is related with S2 using equilibrium theory as 0.27 times that of S2 and no phase lag.

- This method of separation is through ‘inference’

# Prediction of tides

- Since tides are highly periodic, they can be predicted. The method involves a summation of all the constituents at a given place by making use of the known amplitudes and phases of these constituents.

- $T(t) = Z0 + \sum Hn\ fn \cos[\sigma nt - gn + (Vn + un)]$
- Where Z0 is the mean sea level, Hn and gn are the amplitude and phase of the n th constituent. Vn adjusts the phases to allow for astronomical conditions at the time of origin of data. Hnfn accounts for the amplitude, with fn and un accounts for the nodal adjustment.

- Admiralty NP 159 method (using M2, S2, K1 and O1 only four major constituents) predictions are made. The rest of the constituents are calculated by ‘inference’

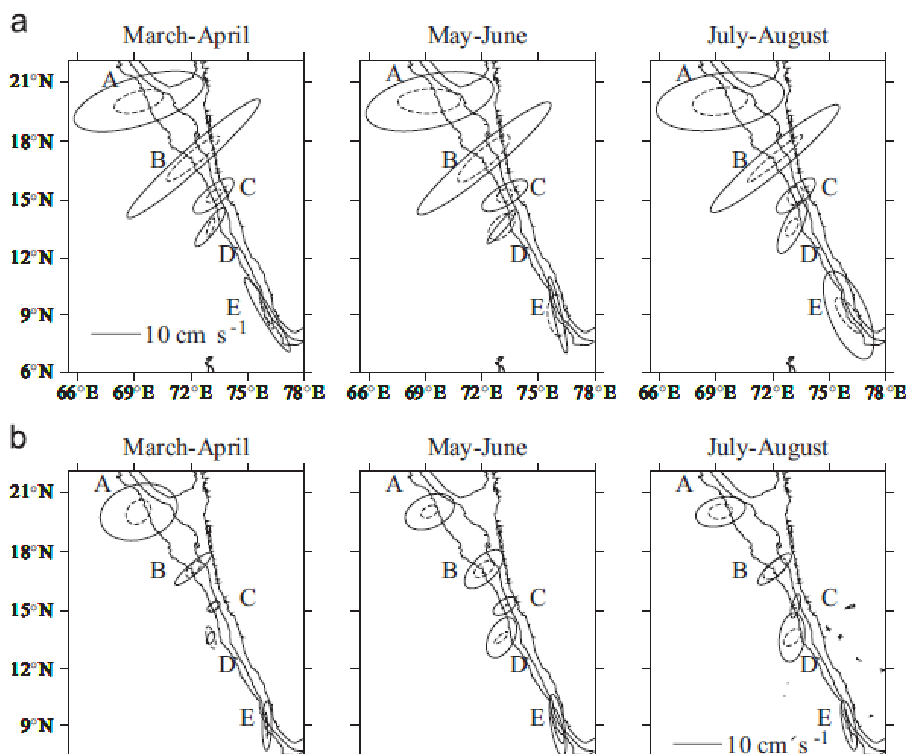
- Various softwares, including ‘TASK’ (Tidal Analysis Software Kit) uses predictions using constituents, that are obtained by analysis of previous records.

# Tide Tables on harmonic constants

- Monaco Tide Tables
- Admiralty Tide Tables, Vol I, II, III
- The above Tables provide information on the amplitudes and phases in standard ports and minor ports worldwide, which can be used for future predictions

# Harmonic analysis of currents

- Usually harmonic analysis of currents is done separately for velocity components. The results are shown in the form of tidal ellipses



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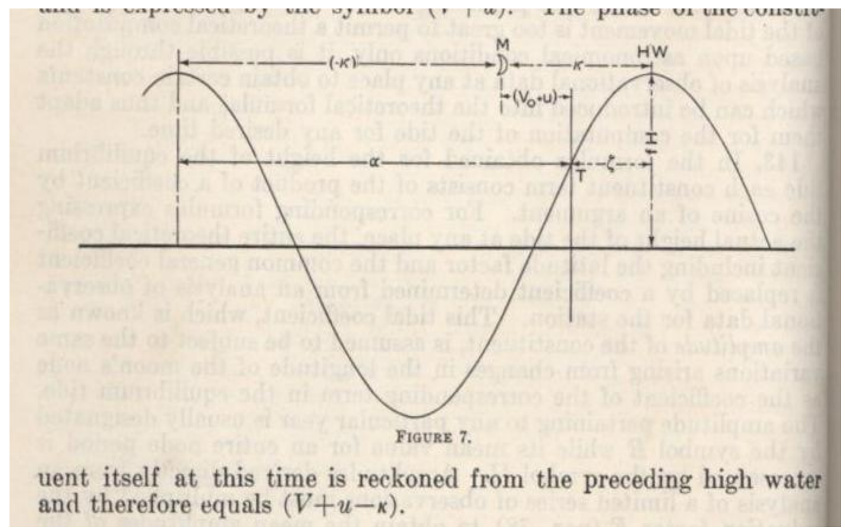


and is expressed by the symbol $(V+u)$. The phase of the constit-

FIGURE 7.

uent itself at this time is reckoned from the preceding high water and therefore equals $(V+u-\kappa)$.